Parabolas and Hyperbolas

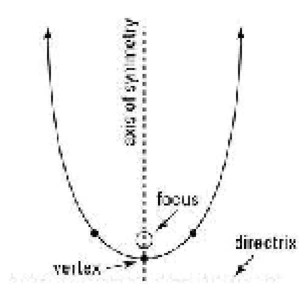
Year 10A

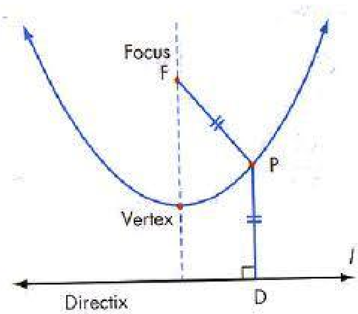
Content Overview

- Define parabola
    - Define focus
    - Define directrix
    - Define axis of symmetry
    - Define vertex
- Solve parabolic equations and plot solution
- Define hyperbola
    - Foci and locus of points
- Solve hyperbolic equations and plot solution

Related Curriculum Statements

Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations (ACMNA267)
using a range of strategies to investigate the effect of multiplying by a constant term, including negative numbers
connecting the graphical and algebraic representations and describing the transformation

Teacher Notes	Classroom Activities
Lesson 1: 60 min	**Introduce, Recall, Inform, Define, Understand**
Parabola: A parabola is a set of points in the plane, that are equidistant from a point called Focus and a line called Directrix. **Vertex:** The vertex is the point on the parabola where the parabola makes its sharpest turn. It is half way between the focus and the directrix. **Axis of symmetry:** If the parabola opens up or down, then the line passing through the vertex of the parabola parallel to y-axis is called axis of symmetry. If the parabola opens right or left, then the line passing through the vertex of the parabola parallel to x-axis is called axis of symmetry. Axis of symmetry of parabola is also called Axis of Parabola. **Focus:** The focus of a parabola is a fixed point on the axis of symmetry at a set distance from the vertex of the parabola. Focus is always inside the Parabola.	**Recall Graphing Quadratic Equations (10 min):** Revise quadratic equations and graphing parabolas by plotting points using quadratic equations. **Understand Vertex, Axis of Symmetry, Focus and Directix of Parabola (25 min):** Have students understand the shape of parabola with the help of a diagram and the terms related to it. Have students understand that the standard form of equation of a parabola is the standard form of the quadratic equation. $y = f(x) = ax^2 + bx + c$ If $a>0$, the parabola opens upward. If $a<0$, the parabola opens downward. Vertex is the minimum or maximum point of the parabola. If the parabola opens up, the vertex is the minimum point and if the parabola opens down, the vertex is the maximum point of the parabola. Vertex can be calculated as $(-b/2a, f(-b/2a))$ 

Directrix: A line perpendicular to the axis of symmetry of the parabola is called directrix. The directrix is outside the parabola and it does not intersect the parabola.	Have students understand that axis of symmetry is a line that passes through the vertex and divides the parabola in to two symmetric parts. The axis of symmetry is the line x= -b/2a. Have students introduce with **Focus of Parabola**. It is a point on the axis of symmetry on a set distance from the vertex. The Focus is always inside the parabola. If the parabola opens up, the focus is above the vertex and if the parabola opens down, the focus is below the vertex. If parabola opens right, then the focus is to the right of the vertex and if the parabola opens left, then the focus is to the left of the vertex. Have students understand that Directrix is a line perpendicular to the axis of symmetry. The directrix is always outside the parabola. If the parabola opens up, the directrix is below the vertex and if the parabola opens down, the directrix is above the vertex. Make it clear to the students that directrix is always on the opposite side of the focus. It means that if the focus is above the vertex, then the directrix is below the vertex. http://www.algebra.com/algebra/homework/equations/EQ.lesson **Relationship Between the Focus and Directrix of Parabola (10 min):** The distance between the focus and the vertex of the parabola is the same as the distance between the vertex and the directrix at the point where directrix intersects the axis of symmetry. Make it clear to the students that distance of each and every point on the parabola from the focus is equal to the length of perpendicular from that point to the directrix.  In this figure FP = PD In other words the ratio of FP and PD is always equal to 1 in parabola. FP/PD = 1 This is true for all the points on the parabola. This ratio is known as eccentricity. http://www.mathwarehouse.com/quadratic/parabola/focus-and-directrix-of-parabola.php http://www.ies.co.jp/math/java/conics/draw_parabola/draw_parabola.html Have students watch this video. http://www.brightstorm.com/math/algebra-2/quadratic-equations-and-inequalities/focus-and-directrix-of-a-parabola/ Watch this video to check out the method of finding vertex, focus and directrix of the parabolic equation given in standard form. http://www.brightstorm.com/math/algebra-2/quadratic-equations-and-inequalities/focus-and-directrix-of-a-parabola-problem-2/ **Convert Standard form of Parabolic Equation to Vertex Form (10 min):**

	The vertex form of the parabolic equations is expressed as, $y = a(x-h)^2 + k$ where (h, k) represents the vertex. If ‘a’ is positive then the parabola opens upward like “U”. If ‘a’ is negative then the parabola opens downward. Have students understand converting standard form of parabolic equations to the vertex form. For example, Convert $f(x) = 2x^2 + 5x – 3$ in Vertex Form. Factor out the coefficient of x^2 i.e. 2, $f(x) = 2(x^2 + 5/2\ x – 3/2)$ Take half of the coefficient of x, square it and add and subtract it inside the brackets. Half of 5/2 is 5/4 and its square will be 25/16. $f(x) = 2(x^2 + 5/2\ x + 25/16 – 25/16 – 3/2)$ $f(x) = 2(x^2 + 5/2\ x + (5/4)^2 – 49/16)$ $f(x) = 2[(x + 5/4)^2 – 49/16]$ $f(x) = 2(x + 5/4)^2 – 49/8$ $f(x) = 2[x – (-5/4)]^2 + (-49/8)$ Which is the required vertex form. Vertex (-5/4, -49/8) Axis of symmetry x = -5/4 http://www.youtube.com/watch?v=pLlXzxTePYI&feature=related Provide students with worksheet to practice problems. Refer to the supplementary document “WORKSHEET YEAR 10A TERM2 LESSON1.DOCX”. **Wrap Up (5 min):** Revisit the lesson. Announce homework and next lesson’s topic. http://www.stewartcalculus.com/data/CALCULUS%206E%20Early%20Transcendentals/upfiles/ess-reviewofconics.pdf
Lesson 2: 60 min	**Understand, Practise, Use, Apply**
Solve parabolic equations and plot solution: 1. Find the vertex. 2. Find axis of Symmetry. 3. Find some points using the step Pattern Method. 4. Plot the points and sketch the parabola. 5. Find focus and plot it. 6. Find equation of directrix and draw it.	**Review (15 min):** Revise Parabola and important terms related to it like vertex, axis of symmetry, focus and directrix. **Solve Parabolic Equations and Plot Solution (40 min):** Have students understand the steps for solving and plotting parabolic equations. 1. Convert into vertex form if the parabolic equation is given in standard form like $y = a(x-h)^2 + k$ 2. Determine how the parabola opens. If $a < 0$ the parabola opens downward and if $a > 0$, the parabola opens upward. 3. Find the Vertex. Vertex of the parabola is (h, k). Plot the vertex. 4. Draw axis of symmetry. It is the line x = h. 5. Find x and y intercepts or Use step pattern Method to find out some points to plot parabola. 6. Plot one or two more points on one side of the axis of symmetry. 7. Use symmetry to plot one or two more points on the other side of the axis of symmetry. 8. Draw the parabola through the points.

9. Find a/4. Let the resulting number is x. Then Plot the focus on the axis of symmetry by counting 'x' points from the vertex inside the parabola.
10. Draw the directrix by counting 'x' points outside the parabola, perpendicular to the axis of symmetry.

Demonstrate the process of graphing parabola on the board using an example.
For example,
Graph the parabola $y = 3(x+1)^2-5$
In this equation a=3>0, so parabola will open upward.

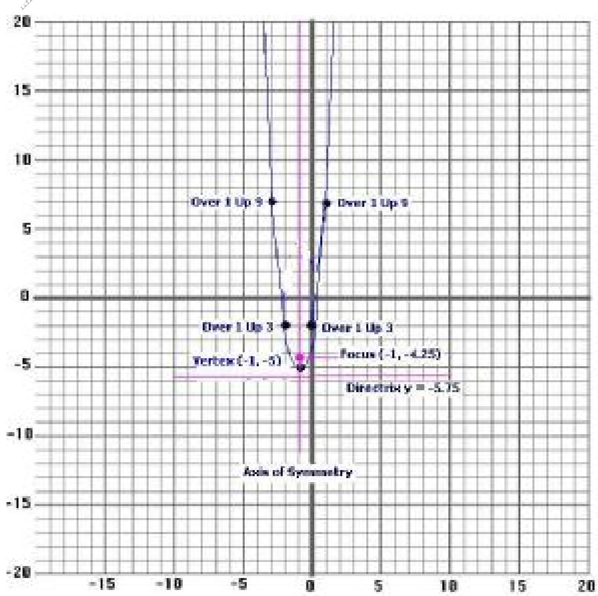
<u>Vertex (h, k)</u>
(-1, -5)
<u>Axis of symmetry</u>
$x = h \Rightarrow x = -1$

<u>Find Points using Step Pattern Method</u>
There is another very easy step pattern method of finding points of parabola which needs no calculations and neither you need to find the x and y intercept. Watch this video to have a clear idea of it.
http://www.youtube.com/watch?v=ZJDM4TE6I30&feature=related

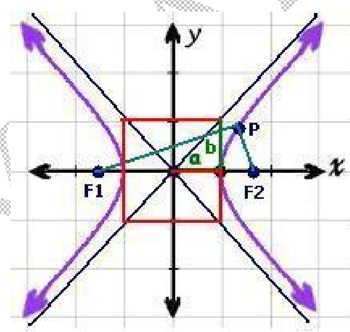
We will use the step pattern (1, 3, 5, 7)*a
Multiply the numbers by a, which is 3. So the steps will be 3, 9, 15, ...
As the parabola is opening upward so Plot the first step 3 by moving one point left from vertex and counting 3 points in upward direction. Now Plot the second step 9 by moving ne point left from the first step and counting 9 in upward direction.
Similarly plot two points on the right side of the vertex using the symmetry of the parabola.
Join the points to make a parabola.

<u>Focus:</u>
a/4 = ¾ = 0.75
Focus will be 0.75 points above the vertex. So It will be (-1, -4.25)

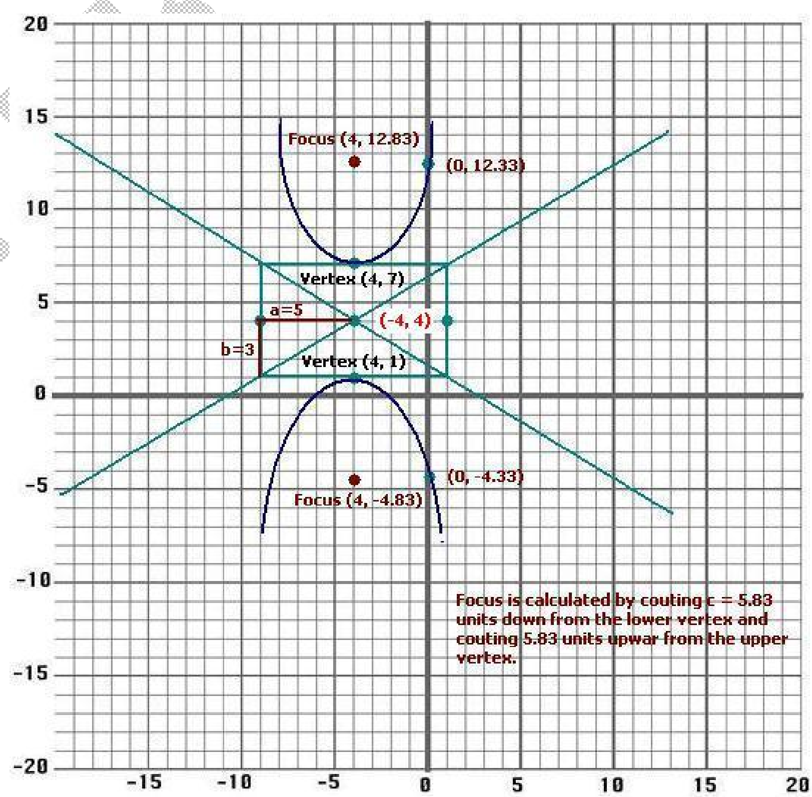
<u>Directrix:</u>
Directrix will be 0.75 points below the vertex.
So equation of directrix will be y = -5.75



Have students watch these videos.

	http://www.youtube.com/watch?v=qq0emjqieQ8&feature=endscreen http://www.youtube.com/watch?v=kRT7quN7uBU Provide students with worksheets to practice plotting Parabolas showing their vertex, axis of symmetry, focus and directrix. Refer to the supplementary document “WORKSHEET YEAR 10A TERM2 LESSON2.DOCX”. **Wrap Up (5 min):** Revisit the lesson. Announce homework and next lesson’s topic.
Lesson 3: 60 min	**Apply, Investigate, Analyse, Experiment, Research**
Hyperbola Hyperbola is the locus (set) points P, such that the difference between the distances from P to foci, F_1 and F_2, are constant. Equation of hyperbola is $(x-h)^2/a^2 – (y-k)^2/b^2 = 1$ Where (h, k) is centre of hyperbola. A hyperbola has two vertices, two foci, two directrices, a centre and two asymptotes. **Foci of Hyperbola:** “Foci” is the plural of focus. Coordinates of foci of hyperbola are (±c, 0). Where $c^2 = a^2 + b^2$	**Review (15 min):** Recall standard and vertex form of the equation of parabola and review the steps to graph a parabola. **Introduction to Hyperbola (30 min):** Draw a diagram of hyperbola on the board and have students understand the definition of hyperbola with the help of the diagram.  Equation of hyperbola is $(x-h)^2/a^2 – (y-k)^2/b^2 = 1$ Where (h, k) is the <ins>**centre**</ins> of the hyperbola. If the centre of hyperbola is at the origin then equation of hyperbola becomes $x^2/a^2 – y^2/b^2 = 1$ <ins>**Vertices**</ins> (±a, 0) <ins>**Foci**</ins> (±c, 0) where $c^2=a^2+b^2$ Equations of asymptotes y = ±(b/a)x Have students understand that an <ins>**asymptote**</ins> is a line that forms a "barrier" to a curve. The curve gets closer and closer to an asymptote, but does not touch it. http://classof1.com/homework_answers/math/hyperbola/ http://www.mathsisfun.com/geometry/hyperbola.html Make it clear to the students that if the centre of the hyperbola is not at the origin then the coordinates of the vertices and foci will be different according to the centre of the hyperbola, they need to be calculated by graph. Also if the equation of the hyperbola is like $(y-k)^2/b^2 - (x-h)^2/a^2 = 1$, then the shape of hyperbola will be up and down instead of left and right. Have Students watch these videos. http://www.brightstorm.com/math/algebra-2/conic-sections/the-hyperbola/ http://www.winpossible.com/lessons/Algebra_2_Graphing_the_equation_of_a_hyperbola.html#Q1

	Converting Hyperbolic Equations in Standard Form (10 min): Have students understand converting hyperbolic equations in standard form using sample equation. For Example $x^2 – y^2 + 8x – 6y -18 = 0$ Combining like terms, $x^2+ 8x – y^2 - 6y -18 = 0$ $x^2+ 8x – (y^2 + 6y) -18 = 0$ Completing square for x and y $x^2+ 8x +16 – (y^2 + 6y +9) -18 = 16-9$ $(x+4)^2 – (y+3)^2 = 7 + 18$ $(x+4)^2 – (y+3)^2 = 25$ Dividing the equation by 25 $(x+4)^2/25 – (y+3)^2/25= 25/25$ $(x+4)^2/25 – (y+3)^2/25= 1$ Provide students with worksheets to practice problems. Refer to the supplementary document "WORKSHEET YEAR 10A TERM2 LESSON3.DOCX". **Wrap Up (5 min):** Revisit the lesson. Announce homework and next lesson's topic.
Lesson 4: 60 min	**Reflect, Evaluate, Create, Synthesise, Summarise**
Solve hyperbolic equations and plot solution	**Review (15 min):** Recall the shape of hyperbola and revise finding its centre, vertices, foci and asymptotes. **Solve Hyperbolic Equations and Plot Solutions:** Have students understand the steps to graph the hyperbola. 1. Find the centre of hyperbola (h, k). 2. Find the values of 'a' and 'b'. 3. Count 'a' units to the left of the centre and plot a point, similarly count 'a' units to the right o the centre and plot another point. 4. Count 'b' units above the centre and plot a point, similarly count 'b' units below the centre and plot another point. 5. Draw a box using these four points. 6. Draw two asymptotes connecting the opposite corners of the box. 7. Determine whether the hyperbola will be in left right shape or up down shape. If in hyperbola equation the negative sign is with 'x' then hyperbola will away from x i.e in up and down direction and if negative sign is with 'y', then the hyperbola will be away from y, i.e. in left and right direction. 8. Find x and y intercepts and plot the real solutions. 9. Draw the hyperbola by choosing the two of the four points as vertices but make sure not to touch the asymptotes. 10. Label the vertices with their coordinates. 11. Calculate 'c' by $c^2=a^2+b^2$ 12. If the hyperbola is in Up Down Direction, then add and subtract 'c' to the y coordinate of the vertices to calculate the coordinates of foci and if the hyperbola is in Left Right direction then add and subtract 'c' to the x coordinate of the vertices. Label the foci with their coordinates. http://tutorial.math.lamar.edu/Classes/Alg/Hyperbolas.aspx#Graph_Hyper_Ex

	1_a Watch this video to grab a super easy method to draw a hyperbola. http://www.youtube.com/watch?v=sjpOvG7icG4&feature=related Demonstrate the whole process with the help of an example. Solve the equation and Graph Hyperbola. $(y-4)^2/25 – (x+4)^2/9 = 1$ Centre of Hyperbola (h, k) = (-4, 4) a=5, b=3 As negative sign is with x, so hyperbola will be away from x-axis in Up down Direction. X-intercepts will not be real because of the negative sign beside it. So calculating y-intercept by putting x=0 in the equation, $(y-4)^2/25 – 16/9 = 1$ $(y-4)^2/25 = 1+16/9$ $(y-4)^2/25 = 25/9$ $(y-4)^2 = 625/9$ $y-4 = \pm 25/3$ $y = 4\pm 25/3$ $y = 37/3, -13/3$ $y \approx 12.33 , -4.33$ So y–intercepts will be (0, -4.33) and (0, 12.33) $C^2 = a^2 + b^2$ $C^2 = 25 + 9 =34$ $C \approx 5.83$  Follow this link for some sample problems. http://home.windstream.net/okrebs/page63.html

	Have students watch this video. http://www.youtube.com/watch?v=Ju5-fVbegI8 http://www.youtube.com/watch?v=hl58vTCqVIY http://www.youtube.com/watch?feature=endscreen&NR=1&v=lGQw-W1PxBE Provide students with worksheets to practice problems. http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Graphing%20and%20Properties%20of%20Hyperbolas.pdf Refer to the supplementary document “WORKSHEET YEAR 10A TERM2 LESSON4.DOCX”. **Wrap Up (5 min):** Revisit the lesson. Announce homework and next lesson’s topic.

Lesson Planner

Lesson	Teacher Notes	Student Activities Resources
1 **Date** **Time**		**<u>Title:</u>**
2 **Date** **Time**		**<u>Title:</u>**
3 **Date** **Time**		**<u>Title:</u>**
4 **Date** **Time**		**<u>Title:</u>**
5 **Date** **Time**		**<u>Title:</u>**